Understanding and Tuning Mass-Interaction Networks Through Their Modal Representation

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ABSTRACT

Sound is all about vibration, and the GENESIS environment provides an efficient way for modeling and simulating complex vibrating structures, enabling to produce rich sounds.

In this paper, we propose an overview of tools recently developed and available within the GENESIS environment, allowing a better understanding on how mass-interaction networks behave and introducing some enhanced tuning of their vibrating properties. All these tools try to address an inherent need of any creative process either in the physical world or in GENESIS, which is to create bidirectional connections between properties of a phenomenon, in our case, audible sounds, and properties of what produced it, here, mass-interaction networks.

For this purpose, we will introduce the topological and modal representations of such mass-interaction networks and appreciate how relevant it can be to switch between these different representations to really apprehend its inner properties and those of the sounds it produces.

1. INTRODUCTION

GENESIS [1] is a musician-oriented software environment for sound synthesis and musical composition. It implements the modular language CORDIS-ANIMA [2] that provides a set of primitive elements, modules satisfying Newtonian physics.

The elementary bricks of CORDIS-ANIMA are distinguished in two categories: the <MAT> modules that designate "matter" being either moving or fixed and the <LIA> modules that designate interactions and allow interconnections between <MAT> modules.

A GENESIS "topological" model is then constructed by connecting these modules to each other into a dedicated 2D space called "the bench", this by direct manipulation (Figure 1). Such a model will be defined by its "topology", meaning its network structuration, by the parameters of each one of the modules it is made of (M for inertia of MAT modules, K for stiffness, Z for damping for LIA modules), and finally by a set of initial conditions carried by MAT modules (position and/or velocity).

Once built and parameterized, the model will be set in motion through an outer excitation, and its movement will be « listened to » (Figure 1).

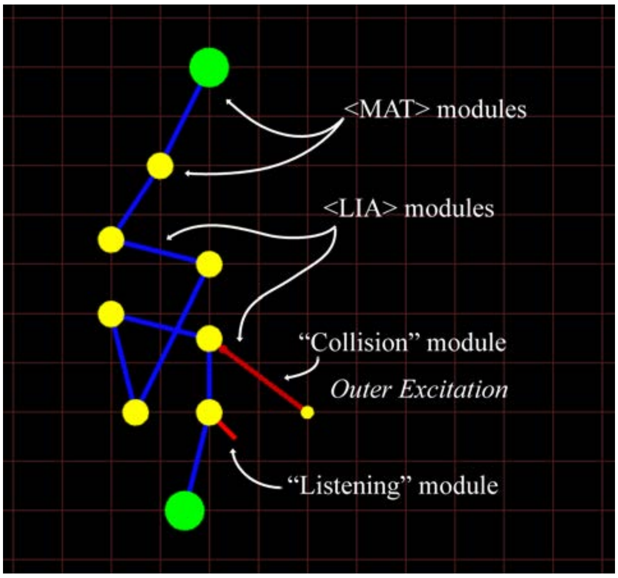

Figure 1. Mass-interaction network of a "string" (indeed, its geometry is quite unusual, but it does not matter. Its topology does). Yellow dots are moving <MAT>, green ones stay fixed.

From there, one can build simple or very large mass-interactions networks, composed with thousands of modules. Once simulated, these allow obtaining complex and rich sounds. But whatever the size of such models, when it comes to exploring the variety of sounds they are able to produce, or even to adjust their nature or structure with the aim of fine-tuning this sounds, a major problem arises. Its widest formulation expressing *a need to establish two-ways relations between causes and effects.*

There are several complementary ways to go on with such concerns.

The first, empirical, is to build one's own knowledge of how objects, models, behave according to how we act on or modify them. One will have to learn how slight adjustment of model parameters will alter perceptive properties of the product of its simulation. Furthermore, in the case of the GENESIS environment, it is now possible to physically experiment virtual models by means of haptic interfaces [3], which allow, by an additional gestural feedback, a very direct and instrumental way to investigate the properties and possibilities of models in terms of musical creation. In both cases, a closed loop is set that involves a model and its simulation, the perceptive outcomes of this simulation, and the adjustment of action or modification applied on this model.

A second one is to get to have a better understanding of the model and the sound it produces by using dedicated analytic technics and tools. These can be derived and adapted from the physical world to the virtual world proposed by GENESIS, or having a more mathematical essence. In our case, we will have to understand the matter at its very smallest scale, so the properties of large objects made of this matter can be easily deducted. For that we will introduce in the following a complementary representation to the topological mass-interaction networks. Based on both representations, and by switching between them, we will present some tools now implemented in the GENESIS environment and allowing to help users in their creative process.

2. MODEL ANALYSIS

Real objects, made out of real matter, and played as musical instruments are since quite recent times under the extensive survey of acoustic, fundamentally motivated in understanding how they produce sound [4]. But even if the mechanisms of some musical instruments are still nowadays freshly discovered, instrument-makers haven't waited to develop tremendous know-how in their conception and neither have the musicians to learn how to play and to take the best out of them.

Sound synthesis by the mean of mass-interaction networks modeling and simulation is one of a few methods that, before producing sounds, require to handle matter and to build complex objects out of it. A virtual matter for virtual objects, of course, but still existing in itself, answering to its own "physics" and that can be studied as it is. And it is precisely by having this objects that one can learn a lot about the sound that it might ultimately produce. This, by developing and using a dedicated "acoustic" of mass-interaction networks

The concept is there to extract, directly by studying the properties of a model, its structure and parameters, without neither simulation nor signal analysis, relevant information regarding to the sound phenomena it will allow to produce. It is basically a transposition of what the Fourier analysis is for the sound signal to the model that enables its simulation.

2.1 The model modal representation

The model modal analysis relies on a mathematical description of the "mechanical network" that is intrinsically every topological CORDIS-ANIMA model. This mechanical network must be a linear vibrating structure moving with only one degree of freedom. Then, the analysis process first aims at switching in representation from a topological model to its equivalent modal model [5]. The latter can be seen as a set of damped harmonic oscillators with unitary inertias (counting as much oscillators than the number of punctual moving <MAT> of the original topological models).

These oscillators represent the stationary vibration modes of the original topological structure. Ultimately, a proper study of the latters will allow a complete knowledge of the original model in terms of mechanical behaviour and acoustical properties.

The following development applies to free vibrating structures without any outer perturbation.

2.2 From topological model to modal model

As a network, a linear topological CORDIS-ANIMA model can be mathematically described by the combination of 3 real symmetric matrices: $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{Z}$, carrying respectively inertia, stiffness and damping parameters of its constitutive modules.

Obtaining the equivalent modal model of a topological one is done by diagonalizing two real symmetric matrices $\mathbf{K}'$ et $\mathbf{Z}'$ in the same basis.

$$\mathbf{Z}' = \mathbf{M}^{-1/2}\mathbf{Z}\mathbf{M}^{1/2} \quad \& \quad \mathbf{K}' = \mathbf{M}^{-1/2}\mathbf{K}\mathbf{M}^{1/2} \qquad (1)$$

This is achievable only if the matrices $\mathbf{K}'$ and $\mathbf{Z}'$ commute (ie : $\mathbf{K}'\mathbf{Z}' = \mathbf{Z}'\mathbf{K}'$), which is always the case if they are linearly co-dependent [6], meaning that $\mathbf{Z}'$ is proportional to $\mathbf{K}'$:

$$\mathbf{Z}' = a + b\mathbf{K}' \quad \text{or} \quad \mathbf{Z} = a\mathbf{M} + b\mathbf{K} \qquad (2)$$

If it answers to relation (2), the viscosity matrix is then said "proportional". The diagonalization process will allow to obtain: two diagonal matrices $\mathbf{K}^{\mathrm{m}}$ and $\mathbf{Z}^{\mathrm{m}}$, respectively containing n modal stiffness $\mathrm{k}_{\mathrm{i}}^{\mathrm{m}}$ and n modal damping $\mathrm{z}_{\mathrm{i}}^{\mathrm{m}}$, and an additional $\mathbf{Q}^{\mathrm{m}}$ matrix such as :

$$\mathbf{Z}^m = \mathbf{Q}^m\mathbf{Z}'\,{}^t\mathbf{Q}^m \quad \text{and} \quad \mathbf{K}^m = \mathbf{Q}^m\mathbf{K}'\,{}^t\mathbf{Q}^m \qquad (3)$$

Mathematically, $\mathrm{k}_{\mathrm{i}}^{\mathrm{m}}$ and $\mathrm{z}_{\mathrm{i}}^{\mathrm{m}}$ are the respective eigenvalues of $\mathbf{K}'$ and $\mathbf{Z}'$, and the $\mathbf{Q}^{\mathrm{m}}$ matrix, representing the common basis of $\mathbf{K}^{\mathrm{m}}$ and $\mathbf{Z}^{\mathrm{m}}$, contains their associated eigenvectors (directly interpretable as modal shapes of the equivalent topological model).

The n harmonic oscillators of the modal model are then directly defined by unitary inertias, $\mathrm{k}_{\mathrm{i}}^{\mathrm{m}}$ and $\mathrm{z}_{\mathrm{i}}^{\mathrm{m}}$ as parameters.

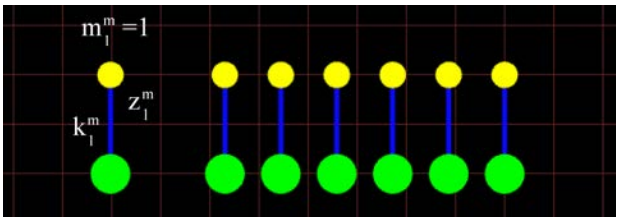

Figure 2. Example of what can be an equivalent Modal Model of (Figure 1) Topological Model.

2.3 Reading a model modal representation

Some of the mathematical tools that are used for describing and studying vibratory and acoustical properties of physical matter are adaptable to the virtual matter handled in GENESIS. Nevertheless, CORDIS-ANIMA relies on a discrete representation of time and matter, which implies well-quantified differences between continuous and discreet equations of movement.

Having the modal model, it is now possible to simply analyze each of its individual elementary harmonics oscillators [7]. They are defined by 3 parameters, m (=1), k_i^m and z_i^m in direct relation with:

The vibration mode frequency:

$$f_i = F_e \cdot \arccos\left(\frac{2-\left(k_i^m + z_i^m\right)}{2\cdot\sqrt{1-z_i^m}}\right) \qquad (4)$$

Where F_e is the sampling rate value of signals produced when simulating a GENESIS model (default value: 44100 Hz)

The vibration mode damping time:

$$\frac{1}{\tau_i} = -\frac{F_e}{2}\cdot\ln\left(1-z_i^m\right) \qquad (5)$$

We can also learn about relative amplitudes of the different oscillators by a proper reading of the transfer matrix $\mathbf{Q}^{\mathrm{m}}$. By choosing the topological structure moving <MAT> on which a listening module will be plugged (identified as α), and the topological model moving <MAT> that will be submitted to an impulsive excitation (identified as β). Then we can estimate the relative amplitude of each vibration mode impulse response according to:

$$\mathbf{A}_i = \mathbf{Q}^{\mathrm{m}}_{(\alpha,i)} \cdot \mathbf{Q}^{\mathrm{m}}_{(\beta,i)} \qquad (6)$$

The theory beyond complex excitations and sustained oscillations of mass-interaction networks is already greatly documented [8]. From the modal representation perspective, calculation of each independent harmonic oscillator transfer function enables a first insight of vibrating model properties under such constraint. For instance, resonant frequencies can be determined and a whole frequency response of a topological model can be displayed.

2.4 Model analysis implementation

A specific tool for model modal analysis has been developed in GENESIS [9] and has now been updated. Follows a quick tour of its latest improvements and functionalities.

The diagonalization process involved in switching from topological to modal representations is processed by using the Jacobi method [10, 11], which allows to obtain simultaneously and efficiently both eigenmodes and eigenvalues of a given matrix. This operation is quite costly but enable to compute the modal representation and acoustical properties of models counting more than 9000 moving <MAT> modules.

Regarding the proportionality constraint over the viscosity matrix (2), the modal analysis can be run even if the model is not proportional. However, we chose to implement a simple algorithm testing this proportionality prior to the diagonalization process, which alerts the user about the relative quality of the obtained result.

One can select a model or a portion of it directly from the bench and launch the Modal Analysis function.

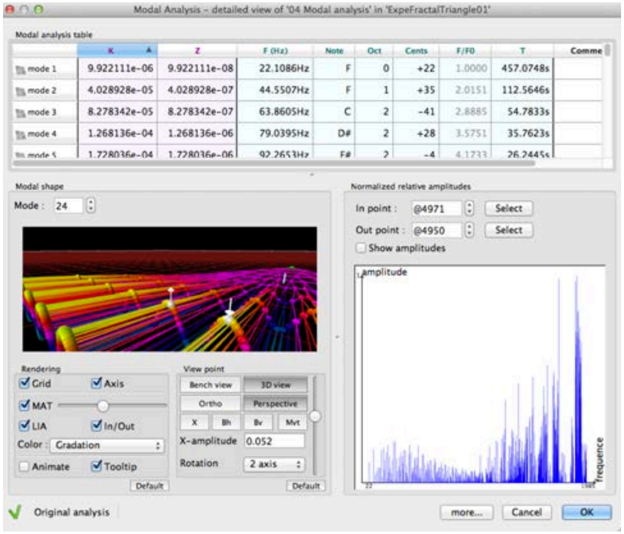

Figure 3. Screenshot of the whole Modal Analysis Window.

The displaying window of the result is organized in 3 distinct parts (Figure 3), which are:

• A general sheet section, gathering the whole information relative to each identified vibration mode of the structure, and by extension, to each partial that will compose a sound produced out of it. It also provides an explicit conversion from frequency to equivalent musical note, octave and cents (Figure 4).

Modal analysis table

	K	Z	F (Hz)	Note	Oct	Cents	F/F0	T
mode 1	2.588438e-07	2.588438e-09	3.5709Hz	---	0	0	1.0000	17520.7873s
mode 2	8.452504e-07	8.452504e-09	6.4528Hz	---	0	0	1.8071	5365.4484s
mode 3	1.995103e-06	1.995103e-08	9.9138Hz	E	-1	-66	2.7763	2273.1391s
mode 4	2.307152e-06	2.307152e-08	10.6610Hz	F	-1	-41	2.9855	1965.6908s
mode 5	3.995598e-06	3.995598e-08	14.0297Hz	A#	-1	-65	3.9289	1135.0359s
mode 6	4.819763e-06	4.819763e-08	15.4089Hz	B	-1	-3	4.3151	940.9483s
mode 7	6.898037e-06	6.898037e-08	18.4341Hz	D	0	+8	5.1623	657.4548s
mode 8	7.586688e-06	7.586688e-08	19.3324Hz	D#	0	-10	5.4139	597.7770s
mode 9	7.940313e-06	7.940313e-08	19.7778Hz	D#	0	+29	5.5386	571.1547s
mode 10	9.908599e-06	9.908599e-08	22.0935Hz	F	0	+21	6.1871	457.6981s
mode 11	1.212960e-05	1.212960e-07	24.4446Hz	G	0	-4	6.8455	373.8908s
mode 12	1.441254e-05	1.441254e-07	26.6458Hz	G#	0	+45	7.4619	314.6667s

Figure 4. Modal Analysis Table, gathering modal model information. Purple columns collect the k_i^m and z_i^m modal parameters. The green columns display their associated acoustical properties.

• A modal shapes visualization section, that allows to individually display an interactive 3D representation of each topological structure modal shapes. The latter can be animated and rendered with various static or dynamic color gradations. All this allowing for example to clearly identify nodes and antinodes of the vibrating structure, and ultimately determine where to act on a structure in accordance to modes/partials that should or should not be predominant (Figure 5).

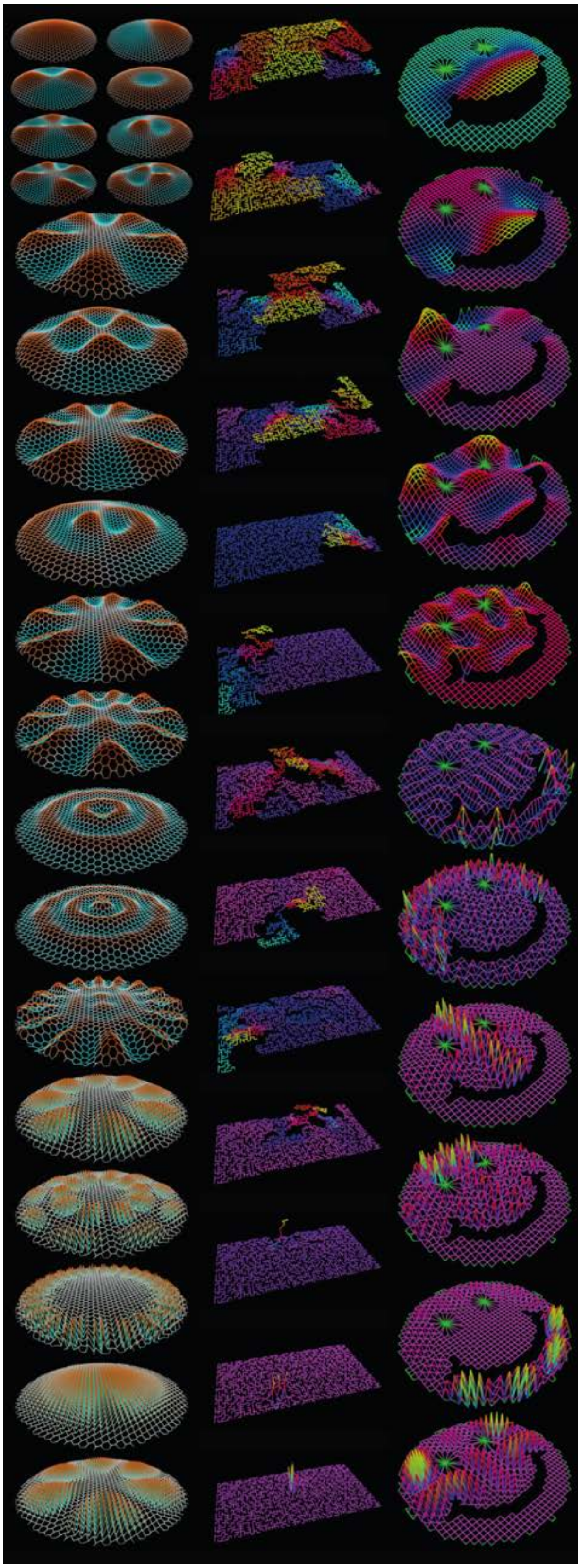

Figure 5. Collection of various topological models modal shapes. Left: we can identify the typical modal shapes of a circular membrane (here composed of 2148 moving elements). Middle: we represent some of the modal shapes of a “labyrinth” model (from which you can escape by looking closely at the first one). Right: various “punk smiley” model modal shapes.

• A normalized impulse response graph section, providing a global picture of the whole frequency spectrum associated with relative amplitudes of model vibration modes. This supposes that both in and out points, respectively, excitation and listening points, are selected directly on the model displayed in the modal shape visualization section (Figure 6).

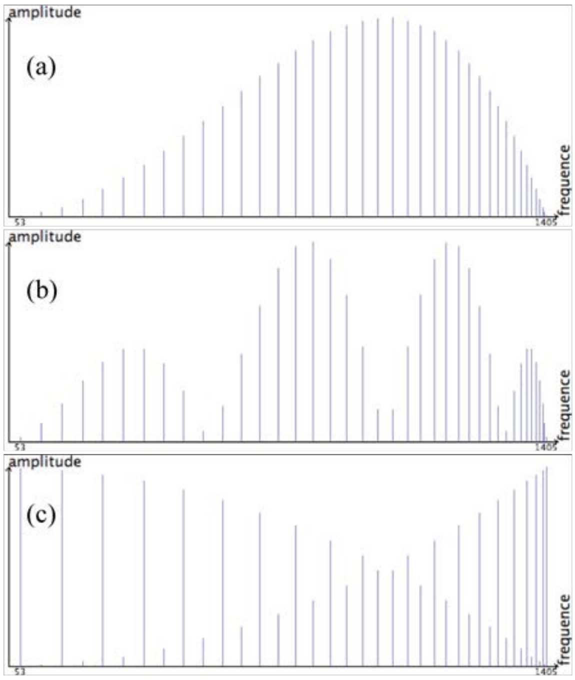

Figure 6. 3 impulse responses of a “string” model, composed of 40 moving <MAT> modules, excited and listened in different points. (a) Input: module 1, Output: module 40. (b) Input: module 4, Output : module 40. (c) Input: module 20, Output : module 20.

3. MODEL TUNING

Having a mass-interaction network and looking to have a better understanding of it is one thing. Trying to tune it, or in the most extreme case, having a “sound” in mind and trying to reproduce it by the mean of mass-interaction networks suggest an other level of considerations. With the idea to provide tools to help the users, and to keep close to the illustrative concepts of the model modal analysis, we have tried to explore the way back from a modal model to its topological equivalent.

First thing first, lets assume that one has built a topological model. How can they tune the latter by modifying properties of its modal equivalent?

3.1 From modal back to topological

In the very limited case of a topological model of which the modal equivalent carry a single harmonic oscillator, we can easily reverse the linear equations (3) and (4) and then tune this oscillator by recovering a proper set of stiffness and viscosity parameters with frequency and damping time as inputs.

Considering a modal model with more than one harmonic oscillator, it wouldn’t be possible to tune each one of them independently because of $\mathbf{K}^m$ and $\mathbf{Z}^m$ that must be preserved for the relation (3) to stay true (necessary condition to get back from a modal representation to the original topological representation). Nevertheless it is possible to apply a single multiplicative coefficient to each of these matrices. This authorizes to set one vibration mode frequency and damping time. Of course all the other structure vibration modes will be proportionally tuned. This function exists in GENESIS and is built directly into the modal analysis window. Hence, from the general sheet section, one can simply edit the acoustical properties (frequency, or related musical note) of one vibration mode of the modal representation and tune the topological model.

Relative amplitudes of modes depend on the model properties and for the most part on how/where it is excited and listened. There is no trivial function enabling to tune a model in a way that the sound it produces has a precise amplitude distribution among its partials. For that, one will rely on the modal shapes visualization and on the impulsion response graph of the Modal Analysis window, which might help in finding the best set of input(s)/output(s).

It is not possible to have an extensive control over the physical parameters of a model, according to acoustical properties, without altering the equivalence/bridge standing between its modal and topological representations. But by breaking this relation that allows to keep *causes* and *effects* close together, advanced tuning possibilities can be foreseen.

3.2 From modal back to …?

Indeed, at a certain point, one could seek to obtain a GENESIS model given nothing else that the description of a sound they want it to produce.

A simple solution to that concern would be to recompose complex sounds by reproducing what's done in additive sound synthesis or modal synthesis [12], which as already been specifically studied for CORDIS-ANIMA applications in [5]: Knowing the modal characterization of a sound phenomenon, we could compose it by synthesizing as many sinusoidal signals as necessary, this by replicating a relevant amount of independents harmonic oscillators. The main issue of this approach is that one will have to artificially selectively distribute energy among the independent oscillators and selectively "listen" to each one of them. Thus, it prevents to obtain a topological representation of the proposed modal model and might considerably limit users in theirs exploration process.

In [13], the authors already explore ways to tune a specific model under frequency constraints. Nevertheless a more general approach, aiming at obtaining a topological mass-interaction network out of multiple acoustical descriptors of sounds, is addressed in [14] and is referred as an "inverse problem". Follows the outcomes of preliminary investigations in simplified cases and their effective implementation in the GENESIS environment.

3.3 An inverse problem

A model of n moving <MAT> has an equivalent modal model of n independent oscillators and thus n eigenmodes. If we now consider the description of a sound composed of n partials, setting a modal model of n moving <MAT> as an input, then a generative topological model must have at least n moving <MAT>, and we can suppose it is enough to define it. Among the multiple solutions that this problem can have, it is necessary to arbitrary choose an alleged generative mass-interaction network. The simplest topological model of n moving <MAT> is the linear "chain", a "string" fixed at its extremities (such as in Figure 7), at one extremity or none.

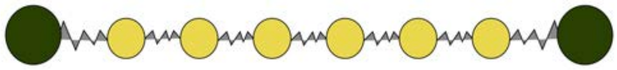

Figure 7. CORDIS-ANIMA linear chain model.

The reduced inverse problem is then to define, given 3 vectors of n frequencies, amplitudes and damping times, a generative mass-interaction string of n moving <MAT> connected by visco-elastic interactions. Different approaches have been followed to resolve it, all basically trying to recover the descripting matrices of the topological ($\mathbf{M}$, $\mathbf{K}$, $\mathbf{Z}$) model given diagonal matrices of the tuned modal model ($\mathbf{K}^m$ and $\mathbf{Z}^m$):

The first one, a numerical resolution, relies on an adapted optimization procedure. It partially addresses the reduced problem by tuning only the stiffness parameters of the generative model, allowing an individual control of each structure vibration mode frequencies (in this case, the moving <MAS> modules are set with unitary inertias).

The second is algebraic and allows to control frequencies as well as relative amplitudes of each vibration mode of the generative structure. Hence, the stiffness and inertia parameters of the latter are tuned. Unlike the numerical approach, this one is case specific and only allows to generate "string" models.

In this case the relative amplitudes are set by direct manipulation of the modal shapes matrix $\mathbf{Q}^m$.

Further details about the previous can be found in [14]

A third approach has been dedicated to enable a specific control over vibration modes damping time. We previously discussed the term of "proportional viscosity matrix" and expressed it with relation (2). Going from a modal representation to a topological one implies to stick close with this rule. Hence, having the inertia and stiffness parameters of a topological model already tuned regarding frequencies and relative amplitudes of its vibration modes, gives a pretty restricted control over the relation (2). Indeed, only two multiplicative values, *a* and *b*, can be edited, meaning that only two vibration modes damping time can be adjusted (the other ones will be proportionally balanced).

Actually, these two values are physically coherent and are a proper metaphor of external and internal viscosities of a mechanical object. *a*, applied to the inertias matrix, can be liken to an external viscosity and represent the dissipative effect of viscous environment in which a model is immersed. *b*, applied to the stiffness matrix, could represent an internal viscosity and stand for the dissipative effect inherent to the model internal physical matter. Thus, by renaming respectively a and b to z^{int} and z^{ext} in (2) we have a direct expression of the topological model viscosity matrix:

$$\mathbf{Z} = z^{\text{ext}}\mathbf{M} + z^{\text{int}}\mathbf{K} \tag{7}$$

Setting the damping-time of the i^{th} and j^{th} modes, with i < j, implies:

$$\tau_{\text{i}} \geq \tau_{\text{j}} \tag{8}$$

The internal and external viscosities constants are then defined by:

$$z^{\text{int}} = \frac{\exp\left(-\frac{2}{\tau_{\text{i}} F_e}\right) - \exp\left(-\frac{2}{\tau_{\text{j}} F_e}\right)}{\mathbf{K}^{\text{m}}_{(\text{j},\text{j})}\text{-}\mathbf{K}^{\text{m}}_{(\text{i},\text{i})}} \tag{9}$$

$$z^{\text{ext}} = \frac{\mathbf{K}^{\text{m}}_{(\text{j},\text{j})}\left(1-\exp\left(-\frac{2}{\tau_{\text{i}} F_e}\right)\right) - \mathbf{K}^{\text{m}}_{(\text{i},\text{i})}\left(1-\exp\left(-\frac{2}{\tau_{\text{j}} F_e}\right)\right)}{\mathbf{K}^{\text{m}}_{(\text{j},\text{j})}\text{-}\mathbf{K}^{\text{m}}_{(\text{i},\text{i})}} \tag{10}$$

It is interesting to point out that the relation (8) guaranties positive internal viscosity values. On another hand, external viscosities might take negative values, which metaphorically would imply that the model environment provides energy instead of absorbing it.
Also, viscosities do have an influence on the vibration mode frequencies, but, they are quite insignificant, and of course this influence has to be evaluated regarding perceptual matters.

This approach works on every linear mass-interaction network, whatever its topology and complexity.

It doesn't seem possible to go further in controlling the damping-time of a modal model vibration modes without breaking the proportionality, altering the topology of the resulting model, or by staying physically coherent and thus being easy to handle by any user.

3.4 Model tuning implementation

3.4.1 PNSL Scripts

The different approaches previously proposed have been implemented thanks to an additional tool included within GENESIS: The *Physics Networks Scripting Language* (PNSL) [1]. This language has been developed to address mass-interaction networks modeling specific needs. For instance, by putting down some lines of script, it allows to create modules, to inter-connect them, to compute and set them with precise parameters, to automate repetitive tasks regarding topological or bench positions matters, to alter selected models, and so on. All this is fully integrated so one can edit and execute scripts directly into the GENESIS environment (Figure 8), and provides another modeling process, complementary to the bench direct manipulation of modules.

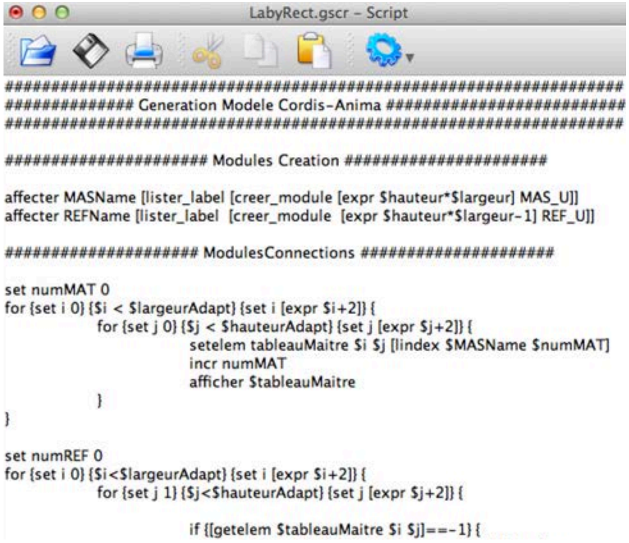

Figure 8. Screenshot of GENESIS PNSL Script Editor.

One of the central considerations in building this tool and its syntax was to make it intelligible and easy to use, even for users non-acquainted with programming languages. Nevertheless, it is built over *Tool Command Language* (TCL) scripting language and therefore includes a lot of advanced libraries [15].

3.4.2 Inverse problems scripts

The numerical approach allows to efficiently obtain generative models defined by up to 20 frequencies. The time of resolution increases dramatically with the number of frequencies to deal with (in the case of 20 tuned vibration modes, the calculation might take about one hour). Its main advantage is that it doesn't depend on the kind of topology you may expect for the output model. It is also convenient to have all inertias set to 1 when bringing the generative model back to a common utilization in using GENESIS.

In its latest version, the algebraic approach is suitable for fixed, half-fixed or free linear chains. Any other topology for the output model would need to have its dedicated algorithm. However, with this approach we managed to obtain generative models defined by up to 50 frequencies and relative amplitudes. The global quality of this resolution, as well as the coherence of the resulting model, relies on the relative proximity of the wanted frequencies. For instance setting the modal model with very close frequencies (of about 1Hz) might introduce numerical imprecisions during the algorithm calculation and lead to partially tuned generative models.

Both of the previous methods can be executed in GENESIS and produce a topological network right on the bench.
The damping-time tuning method needs the user to select a model directly on the bench. Running the script will then only tune its viscosity parameters.

With PNSL, a lot of new functions can be developed, easily used, and of course modified and enhanced, allowing easing prospective works in mass-interaction networks modeling. Inverse problem resolution scripts were the first ones to fully take advantage of this tool, and will be fully accessible as demonstrative scripts in the next update of GENESIS.

4. CONCLUSIONS

The mass-interaction networks modeling used in GENESIS rely on a very unique set of concepts. Its modularity makes it very generative and the exploration of what can be expected out of it seems far from its ending despite decades of developments, experimentations, and of course, of active utilization and artistic creations made by researchers and artists.

Luckily these concepts are close to the physics of real things, so on one hand, users can easily bring their physical instincts in the creative process involved in sound synthesis and musical composition and, on another hand, the tools developed with the aim of easing this creative process can themselves rely on physical metaphors and make direct sense to users.

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